DETERMINING THE SIZE SPECTRUM OF CYLINDRICAL PARTICLES FROM THE LOW-ANGLE INDICATRIX OF LIGHT DISPERSION

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UDC 532.528:7:541.182.3

Formulas are derived for calculating the size spectrum of cylindrical particles by the low-angle method, and the accuracy of this method is evaluated.

A theory and experimental methods of determining the size spectrum of spherical particles in a suspension have been developed in [1-4]. In many instances, however, it is necessary to determine the size spectrum of particles with other than spherical shapes.

We will determine the spectrum of cylindrical particles in a suspension where the cylinder length l can be expressed as a function of the cylinder radius a:

$$l = l(a).$$

The problem reduces to solving the integral equation

$$I(\beta) = I_0 \int_0^{\infty} N(a)\, l(a)\, g(a, \beta)\, da \tag{1}$$

for $N(a)$.

The diffraction of rays at a cluster of cylindrical particles oriented at random in space follows the law [5]:

$$I(\beta) = I_0 k l a^2 \frac{E^2(ka\beta)}{2\pi\beta}, \tag{2}$$

where

$$E(ka\beta) = \sqrt{\frac{\pi}{2ka\beta}}\, J_{1/2}(ka\beta).$$

We introduce

$$\rho = \frac{2\pi a}{\lambda}, \tag{3}$$

and let

$$l/a = m; \tag{4}$$

so that

$$I_{\mathrm{n}} = \frac{1}{\beta^2} \int_0^{\infty} m J^2_{1/2}(\rho\beta)\, N(\rho)\, \rho^2 d\rho, \tag{5}$$

where

$$I_{\mathrm{n}} = 4I(\beta)/k^2 I_0. \tag{6}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 22, No. 5, pp. 890-893, May, 1972. Original article submitted March 31, 1971.

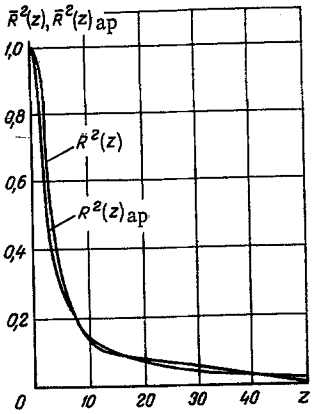

Fig. 1. Function $\bar{R}^2(z)$ and the approximating function $\bar{R}^2_{ap}(z)$ as functions of the parameter $z = k\rho\beta$ (all quantities dimensionless).

We now define

$$\Phi(\rho) = N(\rho)\rho^2 m. \tag{7}$$

For solving the integral equation (5) we will use the Titchmarsch transformation [6]. Then

$$\Phi(\rho) = -c\int_0^\infty \frac{d}{d\beta}(\beta^3 I_n) F(\rho\beta)\, d\beta, \tag{8}$$

where

$$F(\rho\beta) = J_{1/2}(\rho\beta)\, Y_{1/2}(\rho\beta)\, \rho\beta, \tag{9}$$

$$c = \left(\frac{2\pi}{\lambda}\right)^3. \tag{10}$$

From formula (8) we can determine $mN(a)$. With m known, the distribution function density can also be found; in this case

$$\Phi(a) = alN(a). \tag{11}$$

In the case of soft cylindrical particles, the intensity of dispersed radiation is described by the well known Debye equation

$$I = I_0 \frac{1+\cos^2\beta}{2} \cdot \frac{V^2}{\lambda^4}\alpha^2 \bar{R}^2(z), \tag{12}$$

where

$$\bar{R}^2(z) = \frac{1}{z}\int_0^{2z} \frac{\sin\omega}{\omega}\, d\omega - \left(\frac{\sin z}{z}\right)^2. \tag{13}$$

Within the range of values z from 0 to 50 one may approximate function $\bar{R}^2(z)$ by $\bar{R}^2_{ap}(z)$:

$$\bar{R}^2_{an}(z) = \exp[-0.35z] - 0.003z + 0.15. \tag{14}$$

Function $\bar{R}^2(z)$ and the approximating function $\bar{R}^2_{ap}(z)$ are both shown in Fig. 1.

For a polydisperse suspension one may write

$$\hat{I}(\beta) = \int_0^\infty a^6\{\exp[-0.35z] - 0.003z + 0.15\} N(a)\, da. \tag{15}$$

We note that $\hat{I} = (2I/I_0)/(\pi^2 m^2 \alpha^6 (1 + \cos^2\beta))$. Then, after simple transformations, we obtain

$$\hat{I}(\beta) = \int_0^\infty a^6 N(a)\exp[-0.35z]\, da - 0.003k\beta n\int_0^\infty N(a)\, a^7 da + 0.15\int_0^\infty N(a)\, a^6 da \tag{16}$$

or

$$\hat{I}(\beta) = \int_0^\infty a^6 N(a)\exp[-0.35z]\, da - A\beta + C, \tag{17}$$

where A and C are constants.

The first term in expression (16) represents the well known Laplace integral. It is easy to see that this integral is convergent. The magnitude of the first term decreases as β increases, and it becomes a linear function of β at large angles β:

$$-\hat{I}(\beta) = -A\beta + C. \tag{18}$$

We will show that for a γ-distribution

$$N(a) = a^b \exp[-ca], \tag{19}$$

where b and c are distribution parameters.

Inserting (19) into (16) and utilizing the properties of the Γ-function, we have

$$\text{for } \lambda = 0.63\ \mu\text{m} \quad z = 5ma\beta, \tag{20}$$

$$\hat{I}(\beta) = \frac{\Gamma(b+7)}{(5n\beta + c)^{b+7}} - 0.015\frac{\Gamma(b+8)}{c^{b+8}} + 0.15\frac{\Gamma(b+7)}{c^{b+7}} \tag{21}$$

and the variable part of (21) is

$$\mathrm{Var}\,\hat{I}(\beta) = \frac{1}{(5n\beta + c)^{b+7}} - \frac{0.015m(b+7)}{c^{b+8}}\beta + \frac{0.15}{c^{b+7}}. \tag{22}$$

Numerical calculations have shown that $\mathrm{Var}\,\hat{I}(\beta)$ represents the major part of (16) within angles from 0 to 0.1 rad. The computations were made with the following values of the distribution parameters: 1) b = 2, c = 2, m = 10 and 2) b = 2, c = 4, m = 10. Analogous results were obtained from an analysis of the Junge distribution.

We now introduce

$$\hat{I}_1(\beta) = \hat{I}(\beta) - (A\beta + C), \tag{23}$$

$$\Phi(a) = N(a)a^6 \tag{24}$$

with

$$\hat{I}_1(\beta) = \int_0^\infty \Phi(a)\exp[-\xi a\beta]\,da, \tag{25}$$

$$I_{11}(\beta) = \int_0^\infty P(\xi_1)e^{-\xi_1\beta}\,d\xi_1, \tag{26}$$

where $\xi_1 = \pi m a/\lambda$.

We then invert the integral equation (26) with the aid of the Mellin transform [7]:

$$\begin{gathered} \bar{f}(s) = \int_0^\infty f(\xi_1)e^{-S\xi_1}d\xi_1, \\ \varphi(L) = \frac{1}{2\pi i}\int_{r-i\infty}^{r+i\infty} \bar{f}(S)e^{S\xi_1}\,dS. \end{gathered} \tag{27}$$

On the basis of (27) we have now

$$\Phi(a) = \frac{1}{2\pi i}\int_{r-i\infty}^{r+i\infty} I_{11}(\xi\beta)\exp[\xi\beta]\,d\beta. \tag{28}$$

Considering that $\Phi(a) = N(a)a^6$, (28) yields the sought distribution density of soft cylindrical particles.

NOTATION

a	is the cylinder radius;
l	is the cylinder length;
$I(\beta)$	is the angular-distribution function of dispersed light intensity, determined experimentally;
β	is the dispersion angle;
I_0	is the incident light intensity;
k	is the wave number;
λ	is the radiation wavelength;
$m = l/a$;	
$g(a, \beta)$	is the kernel of the integral equation, determined by the optical properties of a particle;
n	is the refractive index of a particle;
$J_{1/2}(x)$, $Y_{1/2}(x)$	are the half-order Bessel functions of the first and of the second kind respectively;
$E^2(z)$, $\bar{R}^2(z)$	are the dispersion efficiency factors;
$\bar{R}^2_{ap}(z)$	is the approximation to function $\bar{R}^2(z)$;
$N(a)$	is the density of particle distribution function;
$V = \pi a^2 l$	is the particle volume;
s, L	are the Laplace-transform operators.

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